

Application of Semi-Latin rectangles designs in maize experiments

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Abstract Semi-Latin rectangles (SLR) find utility in diverse agricultural and industrial experiments where one effect can be attributed to columns and another to rows, with each intersection accommodating 3 experimental units. This article delves into the application of SLR designs in maize experiments and outlines methods for obtaining SLR layouts with a cell size of 3. The proposed approach facilitates the systematic arrangement of experimental treatments to effectively study the effects of interest within the experimental setup.

Keywords: Blocking · Orthogonality · Row-column design · Semi Latin rectangle

Introduction

In agricultural experiments, experimental units often exhibit heterogeneity in two-way cross classified situations that can influence the response variable. To address this, various Row-Column designs (RCDs) have been developed, such as the Latin square design (LSD), Youden square design (YSD), generalized Youden design (GYD), and other related designs. For detailed information on these designs, refer to Shah and Sinha (1996).

Many traditional row-column designs feature only one unit in each row-column intersection. However, certain experimental scenarios require multiple units per intersection. To accommodate such situations, semi-Latin square designs were introduced. A semi-Latin square design is an $r \times r$ array where each row-column intersection contains the same number of experimental units, denoted as k (>1), and each treatment appears exactly once in each row and each column. For detailed insights, one can see Bailey (1988 & 1992); Bailey and Chigbu (1997); Bailey and Royle (1997); Bedford and Whitaker (2001); Chigbu (2003); Parsad (2006); Soicher (2012 & 2013); Bailey and Soicher (2021).

Semi-Latin squares are particularly useful when the number of levels of both nuisance factors (sources of heterogeneity) is the same. However, these designs are not suitable when the number of levels of the nuisance factors differs. An extension of semi-Latin squares to accommodate unequal numbers of rows and columns leads to semi-Latin rectangles. A semi-Latin rectangle is a row-column design involving v treatments, R rows, and C columns, with a cell size of k (>1). Each treatment appears r_r times in each row and r_c times in each column, forming a versatile and adaptable experimental design framework. For theoretical discussions and applications of these designs, readers can refer to Bailey and Monod (2001); Uto and Bailey (2020 & 2022); Yadav *et al.* (2024a, 2024b) and related citations.

Semi-Latin rectangle designs are useful in various types of experiments, including agricultural studies such as plant disease experiments, food sensory experiments, glasshouse experiments, and consumer testing experiments. For example, in glasshouse experiments, $v = 6$ treatments such as maize varieties or sowing dates for maize are applied in $R = 2$ rows (East-West lines) and $C = 6$ columns (North-South lines), with each cell (row-

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column intersection) containing $k = 3$ treatments such that each treatment occurs once ($r_R = 1$) in each row and $r_c = 3$ times in each column. The experimental set up is as (Table 1).

Table 1. Semi-Latin rectangle for 6 treatments in glasshouse experiment for maize

Column → Row ↓	Plot1	Plot2	Plot3	Plot4	Plot5	Plot6
Plot1	1,2,4	2,3,5	3,4,6	4,5,1	5,6,2	6,1,3
Plot2	3,5,6	4,6,1	5,1,2	6,2,3	1,3,4	2,4,5



Despite the existing literature on Semi-Latin rectangles, which has predominantly focused on block size two, there is a notable absence of work for block sizes larger than two ($k > 2$). Recognizing the importance of Semi-Latin rectangles (SLRs), we aim to fill this gap by presenting two distinct methods for constructing SLRs with a cell/block size of 3. This work expands the applicability of SLRs to larger block sizes, enhancing their utility in various experimental settings.

Statistical model: The model for the present study is

$$y = \mu 1 + \Delta' \tau + \phi' \rho + \Psi' \gamma + D' \beta + \varepsilon$$

where, $y = hpk \times 1$ vector of observations,

$1 = hpk \times 1$ vector of 1,

$\Delta' = hpk \times v$ incidence matrix of observations versus treatments,

$\tau = v \times 1$ vector of treatment effects,

$\phi' = hpk \times h$ incidence matrix of observations versus rows,

$\rho = h \times 1$ vector of row effects,

$\Psi' = hpk \times p$ incidence matrix of observations versus columns,

$\gamma = p \times 1$ vector of column effects,

$D' = hpk \times hp$ incidence matrix of observations versus blocks/cells,

$\beta = hp \times 1$ vector of block effects,

$\varepsilon = hpk \times 1$ vector of random errors with Expectation (E)=0 and Dispersion (E) = $\sigma^2 I$,

$(\cdot)' =$ Transpose of matrix

Analysis of the current experimental situations is similar to analysis procedure of Generalized Row-Column design.

Obtaining the layout of designs for experiments

In the current investigation, we are developing the layout of Semi-Latin rectangle (SLR) designs with a block/cell

size of three. We have developed two distinct methods tailored for different scenarios, one for even values of v and another for odd values of v . The detailed methodologies for these approaches are outlined in the following sections.

Method 1: Layout of SLR (when v is odd & multiple of 3)

The current approach for obtaining Semi-Latin rectangle (SLR) designs is centered around the concept of generating the design from an initial column. Once the initial column has been obtained, the entire design can be generated using a specific procedure. Step 1 outlines the method to acquire the initial column, while Step 2 details the procedure for generating the entire design from this initial column, as described in the following sections.

Step-1: First construct a matrix of order $v/3 \times 3$ such that each row sum is constant which is $3(v+1)/2$, where $v = 3n$, $n \geq 3$ is an odd number. First column of the such matrix is obtained by arranging the numbers from up to down as $1, 2, \dots, v/3$ and second column obtained by arranging the numbers $(v/3+1), (v/3+2), \dots, 2v/3$ starting from down to up with leaving a blank in alternative place and after that filling the blank in same way; lastly we obtained the third column by arranging the numbers $(2v/3+1), (2v/3+2), \dots, v$ starting from down to up leaving first space as blank and then alternatively leaving blank place and after that filling the blank in the same way.

For example, for $n = 5$, $v = 15$; matrix of order 5×3 will

$$be \begin{pmatrix} 1 & 8 & 15 \\ 2 & 10 & 12 \\ 3 & 7 & 14 \\ 4 & 9 & 11 \\ 5 & 6 & 13 \end{pmatrix}$$

Step-2: Considering the matrix of order $v/3 \times 3$ which is obtained in step-1 as first column (initial column) of size three and adding one to each element with modulo v we got total v column of size three. The resulting design is Semi-Latin Rectangle of block size three.

By following these steps, we can obtain the layout of Semi-Latin rectangle designs of block size three for the parameters

$$v = 3n \text{ (} n \text{ is odd integer number), } R = n, C = 3n, k = 3, r_R = 1, r_c = 3$$

Examples 1: For $v = 9$, $R = 3$, $C = 9$, $k = 3$, $r_R = 1$, $r_c = 3$ we can obtain a layout of semi-Latin rectangle design as shown below:

Row/col	1	2	3	4	5	6	7	8	9
1	1, 5, 9	2, 6, 1	3, 7, 2	4, 8, 3	5, 9, 4	6, 1, 5	7, 2, 6	8, 3, 7	9, 4, 8
2	2, 6, 7	3, 7, 8	4, 8, 9	5, 9, 1	6, 1, 2	7, 2, 3	8, 3, 4	9, 4, 5	1, 5, 6
3	3, 4, 8	4, 5, 9	5, 6, 1	6, 7, 2	7, 8, 3	8, 9, 4	9, 1, 5	1, 2, 6	2, 3, 7

Examples 2: For $v = 15$, $R = 5$, $C = 15$, $k = 3$, $r_R = 1$, $r_c = 3$ we can obtain a layout of semi-Latin rectangle design as shown below:

Row/col	1	2	3	4	5	6	7	8
1	1,8,15	2,9,1	3,10,2	4,11,3	5,12,4	6,13,5	7,14,6	8,15,7
2	2,10,12	3,11,13	4,12,14	5,13,15	6,14,1	7,15,2	8,1,3	9,2,4
3	3,7,14	4,8,15	5,9,1	6,10,2	7,11,3	8,12,4	9,13,5	10,14,6
4	4,9,11	5,10,12	6,11,13	7,12,14	8,13,15	9,14,1	10,15,2	11,1,3
5	5,6,13	6,7,14	7,8,15	8,9,1	9,10,2	10,11,3	11,12,4	12,13,5

Row/col	9	10	11	12	13	14	15
1	9,1,8	10,2,9	11,3,10	12,4,11	13,5,12	14,6,13	15,7,14
2	10,3,5	11,4,6	12,5,7	13,6,8	14,7,9	15,8,10	1,9,11
3	11,15,7	12,1,8	13,2,9	14,3,10	15,4,11	1,5,12	2,6,13
4	12,2,4	13,3,5	14,4,6	15,5,7	1,6,8	2,7,9	3,8,10
5	13,14,6	14,15,7	15,1,8	1,2,9	2,3,10	3,4,11	4,5,12

Method 2: Layout of SLR (when v is even & multiple of 3)

The process of obtaining the layout of Semi-Latin rectangle (SLR) designs revolves around generating the design starting from an initial column. Once the initial column is obtained, the entire design can be systematically obtained. Step 1 explains how to obtain the initial column, while Step 2 delineates the procedure for generating the complete design from this initial column, as elaborated in the subsequent sections.

Step-1: First construct a matrix of order $v/3 \times 3$ such that difference of the elements in rows are $\pm 1, \pm 2, \pm 3$; here $v = 3n$, n is even number. First row of such matrix is (1, 2, 4). For obtaining first column add 2 in 1, then in resulting add 4 after that add 2 then 4 and so on until we get $(v-3)$. And for obtaining second column add 3 in 2 after that add 3 in resulting until we got $(v-1)$. Lastly for obtaining third column add 2 in 4, then add 4 in resulting after that add 2 and 4 continuously until v came.

For example; for $n = 4$, $v = 12$; matrix of order 4×3

$$\text{will be } \begin{pmatrix} 1 & 2 & 4 \\ 3 & 5 & 6 \\ 7 & 8 & 10 \\ 9 & 11 & 12 \end{pmatrix}$$

Step-2: Considering the matrix of order $v/3 \times 3$ which is obtained in step-1 as first column of size three and adding one to each element with modulo v we get total v column of size three. The resulting layout of design is Semi-Latin Rectangle of block size three.

By following these steps, we can obtain the layout of Semi-Latin rectangle designs of block size three for the parameters

$v = 3n$ (n is an even integer number), $R = n$, $C = 3n$, $k = 3$, $r_R = 1$, $r_c = 3$

Example 3: For $v = 6$, $R = 2$, $C = 6$, $k = 3$, $r_R = 1$, $r_c = 3$ we can obtain a layout of semi-Latin rectangle design as shown below:

Row/col	1	2	3	4	5	6
1	1, 2, 4	2, 3, 5	3, 4, 6	4, 5, 1	5, 6, 2	6, 1, 3
2	3, 5, 6	4, 6, 1	5, 1, 2	6, 2, 3	1, 3, 4	2, 4, 5

Example 4: For $v = 12$, $R = 4$, $C = 12$, $k = 3$, $r_r = 1$, $r_c = 3$ we can obtain a layout of semi-Latin rectangle design as shown below

Row/col	1	2	3	4	5	6
1	1, 2, 4	2, 3, 5	3, 4, 6	4, 5, 7	5, 6, 8	6, 7, 9
2	3, 5, 6	4, 6, 7	5, 7, 8	6, 8, 9	7, 9, 10	8, 10, 11
3	7, 8, 10	8, 9, 11	9, 10, 12	10, 11, 1	11, 12, 2	12, 1, 3
4	9, 11, 12	10, 12, 1	11, 1, 2	12, 2, 3	1, 3, 4	2, 4, 5
Row/col	7	8	9	10	11	12
1	7, 8, 10	8, 9, 11	9, 10, 12	10, 11, 1	11, 12, 2	12, 1, 3
2	9, 11, 12	10, 12, 1	11, 1, 2	12, 2, 3	1, 3, 4	2, 4, 5
3	1, 2, 4	2, 3, 5	3, 4, 6	4, 5, 7	5, 6, 8	6, 7, 9
4	3, 5, 6	4, 6, 7	5, 7, 8	6, 8, 9	7, 9, 10	8, 10, 11

Concluding remarks

The proposed layout of designs is particularly valuable in experimental scenarios characterized by two-way variations, where each intersection requires 3 experimental units. This design framework is especially applicable in maize experiments conducted in controlled glasshouse environments. Additionally, these designs are well-suited for scientific experiments involving maize cultivation in hilly terrains, such as terrace cultivation, where the terraced layout can exhibit two-way variations attributable to differences in soil texture or fertility across different terrace sections. This approach enables researchers to effectively explore and analyze the impact of various factors on maize growth and development under diverse environmental conditions.

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